Unstable particle's wave-function renormalization prescription

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We strictly define two set Wave-function Renormalization Constants (WRC) under the LSZ reduction formula for unstable particles at the first time. Then by introducing antiparticle's WRC and the CPT conservation law we obtain a new wave-function renormalization condition which can be used to totally determine the two set WRC. We calculate two physical processes to manifest the consistence of the present wave-function renormalization prescription with the gauge theory in standard model. We also prove that the conventional wave-function renormalization prescription which discards the imaginary part of unstable particle's WRC leads to physical amplitude gauge dependent.

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I. INTRODUCTION

Wave-function or field renormalization prescriptions have been present for a long time, but at present they encounter some problems for unstable particles [1, 2, 3]. Since there is imaginary part present in unstable particle's self energy, how to deal with it in determining unstable particle's WRC becomes an inevitable problem. We begin our discussion from the fermion field renormalization prescriptions. The fermion Field Renormalization Constants (FRC) (which serve as fermion WRC) can be introduced as [2]

$$\Psi_{0i} \,=\, \sum_j Z_{ij}^{\frac{1}{2}}\Psi_j \,, \qquad \bar{\Psi}_{0i} \,=\, \sum_j \bar{\Psi}_j \bar{Z}_{ji}^{\frac{1}{2}} \,, \tag{1}$$

with

$$Z_{ij}^{\frac{1}{2}} \,=\, Z_{ij}^{L\frac{1}{2}}\gamma_L + Z_{ij}^{R\frac{1}{2}}\gamma_R \,, \qquad \bar{Z}_{ij}^{\frac{1}{2}} \,=\, \bar{Z}_{ij}^{L\frac{1}{2}}\gamma_R + \bar{Z}_{ij}^{R\frac{1}{2}}\gamma_L \,, \tag{2}$$

where i and j are the fermion generations, and γ_L and γ_R are the left- and right- handed helicity operators. Because the bare fermion fields have the relationship $\bar{\Psi}_{0i} = \Psi_{0i}^{\dagger}\gamma^0$, the fermion FRC seem to satisfy the 'pseudo-hermiticity' relationship [2, 3]

$$\bar{Z}_{ij}^{\frac{1}{2}} \,=\, \gamma^0 Z_{ij}^{\frac{1}{2}\dagger}\gamma^0 \,. \tag{3}$$

It's well known that the conventional field renormalization prescription for fermions is

$$\begin{aligned} &\hat{\Gamma}_{ij}(p)\, u_j(p)|_{p^2=m_j^2} \,=\, 0 \,, && \bar{u}_i(p)\hat{\Gamma}_{ij}(p)|_{p^2=m_i^2} \,=\, 0 \,, \\ &\lim_{p^2\to m_i^2} \frac{\not{p}+m_i}{p^2-m_i^2}\hat{\Gamma}_{ii}(p)\, u_i(p) \,=\, u_i(p) \,, && \lim_{p^2\to m_i^2} \bar{u}_i(p)\hat{\Gamma}_{ii}(p)\frac{\not{p}+m_i}{p^2-m_i^2} \,=\, \bar{u}_i(p) \,, \end{aligned} \tag{4}$$

where $\hat{\Gamma}_{ij}$ is the renormalized fermion two-point function. Of course there is no problem for stable fermions, but for unstable fermions Eqs.(4) must be revised since the imaginary part coming from the branch cut of the fermion self energy makes Eq.(3) and Eqs.(4) cannot be simultaneously satisfied [2, 3]. The acceptable fermion field renormalization prescription under the constraint of Eq.(3) should be [4]

$$\begin{aligned} &\tilde{Re}\,\hat{\Gamma}_{ij}(p)\, u_j(p)|_{p^2=m_j^2} \,=\, 0 \,, && \tilde{Re}\,\bar{u}_i(p)\hat{\Gamma}_{ij}(p)|_{p^2=m_i^2} \,=\, 0 \,, \\ &\lim_{p^2\to m_i^2} \frac{\not{p}+m_i}{p^2-m_i^2}\tilde{Re}\,\hat{\Gamma}_{ii}(p)\, u_i(p) \,=\, u_i(p) \,, && \lim_{p^2\to m_i^2} \bar{u}_i(p)\tilde{Re}\,\hat{\Gamma}_{ii}(p)\frac{\not{p}+m_i}{p^2-m_i^2} \,=\, \bar{u}_i(p) \,, \end{aligned} \tag{5}$$

where $\tilde{Re}$ takes the left part of the self energy after removing the branch cut of it.

But such fermion field renormalization prescription makes physical amplitude gauge-parameter dependent (see Ref.[2, 5] and the discussion below). The only way to solve this problem is to discard the constraint of Eq.(3) for

unstable external-line fermions of S-matrix, since Eq.(3) has been broken by the branch cut of unstable fermion's self energy [2]. Under this prescription for diagonal fermion FRC D. Espriu et al. obtain [2]

$$\begin{aligned}
\delta\bar{Z}^L_{ii} &= -\Sigma^L_{ii}(m_i^2) - m_i^2\frac{\partial}{\partial p^2}\left[\Sigma^L_{ii}+\Sigma^R_{ii}+2\Sigma^S_{ii}\right]_{p^2=m_i^2} - \frac{\alpha_i}{2}\,,\\
\delta\bar{Z}^R_{ii} &= -\Sigma^R_{ii}(m_i^2) - m_i^2\frac{\partial}{\partial p^2}\left[\Sigma^L_{ii}+\Sigma^R_{ii}+2\Sigma^S_{ii}\right]_{p^2=m_i^2} - \frac{\alpha_i}{2}\,,\\
\delta Z^L_{ii} &= -\Sigma^L_{ii}(m_i^2) - m_i^2\frac{\partial}{\partial p^2}\left[\Sigma^L_{ii}+\Sigma^R_{ii}+2\Sigma^S_{ii}\right]_{p^2=m_i^2} + \frac{\alpha_i}{2}\,,\\
\delta Z^R_{ii} &= -\Sigma^R_{ii}(m_i^2) - m_i^2\frac{\partial}{\partial p^2}\left[\Sigma^L_{ii}+\Sigma^R_{ii}+2\Sigma^S_{ii}\right]_{p^2=m_i^2} + \frac{\alpha_i}{2}\,,
\end{aligned}\tag{6}$$

where the one-loop fermion self energy is written as

$$\Sigma_{ii}(\not p) = \not p\gamma_L\Sigma^L_{ii}(p^2) + \not p\gamma_R\Sigma^R_{ii}(p^2) + m_i\Sigma^S_{ii}(p^2)\,,\tag{7}$$

and α_i is an arbitrary coefficient. Since there are arbitrary coefficients in Eqs.(6), the definition of fermion FRC has indetermination. This indetermination doesn't affect the neutral current couplings at one-loop level, but changes the charged current couplings. Generally speaking this indetermination cannot be removed in physical results, for example in the physical result of $W^- \to e^-\bar{\nu}_e$.

Besides the indetermination, there is also an unclear problem in this renormalization prescription. There are two set FRC $Z^{\frac{1}{2}}_{ij}$ and $\bar{Z}^{\frac{1}{2}}_{ij}$ respectively for the incoming and outgoing fermions, but how to determine the incoming and outgoing anti-fermion's FRC hasn't been clearly discussed. In this paper we introduce four set WRC, two set for particles and two set for antiparticles, and discuss how to totally determine them. The arrangement of the contents is as follows. We firstly strictly define the particle's WRC under the LSZ reduction formula [6]. Then we discuss how to introduce the antiparticle's WRC and use them together with the CPT conservation law to obtain a new wave-function renormalization condition to totally determine the WRC. In section 4 we calculate two physical processes to manifest that compared with the conventional one the present wave-function renormalization prescription keeps physical amplitude gauge invariant. Lastly we give our conclusion.

II. DEFINITION OF WAVE-FUNCTION RENORMALIZATION CONSTANTS UNDER THE LSZ REDUCTION FORMULA

Generally speaking there are two ways to determine the WRC: one is to introduce FRC which serve as WRC [1, 2, 3, 4], the other is not to introduce FRC and the WRC are determined by the LSZ reduction formula. At present we only discuss the second prescription.

From the LSZ reduction formula the WRC is the *field strength renormalization factor*. For boson one has [6]

$$\int d^4x\, e^{i\,p_j\cdot x}\cdots\int d^4y\, e^{-i\,p_i\cdot y} <\Omega|T\{\phi_j(x)\cdots\phi_i^\dagger(y)\}|\Omega> \sim \frac{i<\Omega|\phi_j(0)|\lambda_j>}{p_j^2-m_j^2+i\epsilon}\cdots\frac{i<\lambda_i|\phi_i^\dagger(0)|\Omega>}{p_i^2-m_i^2+i\epsilon}<j\cdots|S|\cdots i>\,,\tag{8}$$

where Ω is the interaction vacuum, T is the time-ordering operator, ϕ_i and ϕ_j are the Heisenberg fields, p_i and p_j are on mass shell, and λ_i and λ_j are the incoming and outgoing boson states of S-matrix elements. We can introduce boson WRC as follows

$$Z_i^{\frac{1}{2}} = <\Omega|\phi_i(0)|\lambda_i>\,,\qquad \bar{Z}_i^{\frac{1}{2}} = <\lambda_i|\phi_i^\dagger(0)|\Omega>\ .\tag{9}$$

We note that the LSZ reduction formula has only been proved for stable particles [6]. Here we will postulate a generalization of the LSZ reduction formula to unstable particles. Under the postulation the following formulas and Eqs.(8,9) also hold true for unstable particles. Note that the ϵ in Eq.(8) isn't infinitesimal any more for unstable particles under the postulation, since it will be proportional to the unstable particle's decay width according to the Breit-Wigner formula [7]. From Eqs.(8,9) the the scalar boson's propagation amplitude in interaction vacuum is

$$\int d^4x\, e^{i\,p\cdot x} <\Omega|T\{\phi_i(x)\phi_i^\dagger(0)\}|\Omega> \sim \frac{i<\Omega|\phi_i(0)|\lambda_i><\lambda_i|\phi_i^\dagger(0)|\Omega>}{p^2-m_i^2+i\epsilon} = \frac{i\,Z_i^{\frac{1}{2}}\bar{Z}_i^{\frac{1}{2}}}{p^2-m_i^2+i\epsilon}\,.\tag{10}$$

Form Eq.(10), Eq.(8) can also be written as

$$\int d^4x\, e^{i\,p_j\cdot x}\cdots\int d^4y\, e^{-i\,p_i\cdot y} <\Omega|T\{\phi_j(x)\cdots\phi_i^\dagger(y)\}|\Omega>\sim \frac{i\,Z_j^{\frac{1}{2}}\bar{Z}_j^{\frac{1}{2}}}{p_j^2-m_j^2+i\epsilon}\cdots\frac{i\,Z_i^{\frac{1}{2}}\bar{Z}_i^{\frac{1}{2}}}{p_i^2-m_i^2+i\epsilon}\mathcal{M}^{amp}(i\cdots\to\cdots j)\,, \quad (11)$$

where the superscript *amp* represents the amputated Feynman amplitude. From Eqs.(8,9,11) we obtain the familiar result

$$<j\cdots|S|\cdots i>=\;\bar{Z}_j^{\frac{1}{2}}\mathcal{M}^{amp}(i\cdots\to\cdots j)Z_i^{\frac{1}{2}}\,, \quad (12)$$

where the other external-line particle's WRC are ignored for convenience. Thus $Z_i^{\frac{1}{2}}$ is the incoming boson's WRC, and $\bar{Z}_j^{\frac{1}{2}}$ is the outgoing boson's WRC.

From Eq.(10) the boson's WRC can be obtained by expanding the boson propagation amplitude at $p^2\to m_i^2$

$$\frac{i}{p^2-m_i^2-\delta m_i^2+\Sigma_{ii}(p^2)}\sim\frac{i}{(p^2-m_i^2)(1+\Sigma'_{ii}(m_i^2))+\Sigma_{ii}(m_i^2)-\delta m_i^2}=\frac{i\,(1+\Sigma'_{ii}(m_i^2))^{-1}}{p^2-m_i^2+i\,\epsilon}\,, \quad (13)$$

where $\Sigma'_{ii}(m_i^2)=\partial\Sigma_{ii}(m_i^2)/\partial p^2$, and $\epsilon=(1+\Sigma'_{ii}(m_i^2))^{-1}(Im\,\Sigma_{ii}(m_i^2)-i\,Re\,\Sigma_{ii}(m_i^2)+i\,\delta m_i^2)$ is a small quantity which is proportional to the boson's decay width at one-loop level. Therefore one has from Eqs.(10,13)

$$Z_i^{\frac{1}{2}}\bar{Z}_i^{\frac{1}{2}}\;=\;(1+\Sigma'_{ii}(m_i^2))^{-1}\,. \quad (14)$$

On the other hand, from the relationship between the two matrix elements: $<\lambda_i|\phi_i^\dagger(0)|\Omega>=<\Omega|\phi_i(0)|\lambda_i>^\dagger$, we obtain a hermitian conjugation relationship between the incoming and outgoing boson's WRC (see Eqs.(9))

$$\bar{Z}_i^{\frac{1}{2}}\;=\;Z_i^{\frac{1}{2}*}\,. \quad (15)$$

But such hermitian conjugation relationship has been broken by the imaginary parts of unstable boson's propagation amplitudes [5]. This can also been seen from Eq.(14): Eq.(15) requires Eq.(14) must be a real number, but it is obviously not the case for unstable bosons.

For fermion the above discussions become a little more complex. We note that there has been an early discussion of this problem in Ref.[8], in which part of the conclusion will be cited in the following discussions. For the Green function with two fermion external lines we have (like Eq.(8))

$$\int d^4x\, e^{i\,p_j\cdot x}\cdots\int d^4y\, e^{-i\,p_i\cdot y} <\Omega|T\{\psi_j(x)\cdots\bar{\psi}_i(y)\}|\Omega>\sim\frac{i<\Omega|\psi_j(0)|\lambda_{j_s}>}{p_j^2-m_j^2+i\epsilon}<j_s\cdots|S|\cdots i_s>\frac{i<\lambda_{i_s}|\bar{\psi}_i(0)|\Omega>}{p_i^2-m_i^2+i\epsilon}\,, \quad (16)$$

where the subscript s denotes the helicity of the fermion states, and the other boson WRC and propagators have been ignored for convenience. The fermion WRC can be introduced as follows:

$$<\Omega|\psi_i(0)|\lambda_{i_s}>=\;Z_i^{\frac{1}{2}}u_i^s\,,\qquad <\lambda_{i_s}|\bar{\psi}_i(0)|\Omega>=\;\bar{u}_i^s\bar{Z}_i^{\frac{1}{2}}\,, \quad (17)$$

with

$$Z_i^{\frac{1}{2}}\;=\;Z_i^{L\frac{1}{2}}\gamma_L+Z_i^{R\frac{1}{2}}\gamma_R\,,\qquad \bar{Z}_i^{\frac{1}{2}}\;=\;\bar{Z}_i^{L\frac{1}{2}}\gamma_R+\bar{Z}_i^{R\frac{1}{2}}\gamma_L\,. \quad (18)$$

The fermion propagation amplitude thus is

$$\int d^4x\, e^{i\,p\cdot x} <\Omega|T\{\psi_i(x)\bar{\psi}_i(0)\}|\Omega>\sim\frac{\sum_s i<\Omega|\psi_i(0)|\lambda_{i_s}><\lambda_{i_s}|\bar{\psi}_i(0)|\Omega>}{p^2-m_i^2+i\epsilon}=\frac{i\,Z_i^{\frac{1}{2}}\sum_s u_i^s\,\bar{u}_i^s\,\bar{Z}_i^{\frac{1}{2}}}{p^2-m_i^2+i\epsilon}\,. \quad (19)$$

At tree level $\sum_s u_i^s(p)\,\bar{u}_i^s(p)=\not{p}+m_i$. When considered radiative corrections Eq.(19) can become [8]

$$\int d^4x\, e^{i\,p\cdot x} <\Omega|T\{\psi_i(x)\bar{\psi}_i(0)\}|\Omega>\;\xrightarrow{p^2\to m_i^2}\;\frac{i\,Z_i^{\frac{1}{2}}(\not{p}+m_i+ix)\bar{Z}_i^{\frac{1}{2}}}{p^2-m_i^2+i\epsilon}\,, \quad (20)$$

where x is a small radiative correction which is proportional to the fermion's decay width at one-loop level [8]. We note that the result of Eq.(20) is firstly proposed in Ref.[8] as an assumption, here we derive it under a rational foundation. From Eq.(19), Eq.(16) can also be written as

$$\int d^4x\, e^{i\,p_j\cdot x}\cdots\int d^4y\, e^{-i\,p_i\cdot y} <\Omega|T\{\psi_j(x)\cdots\bar{\psi}_i(y)\}|\Omega>\sim \frac{i\,Z_j^{\frac{1}{2}}u_j^s\,\bar{u}_j^s\,\bar{Z}_j^{\frac{1}{2}}}{p_j^2-m_j^2+i\epsilon}\mathcal{M}^{amp}(i_s\cdots\to\cdots j_s)\frac{i\,Z_i^{\frac{1}{2}}u_i^s\,\bar{u}_i^s\,\bar{Z}_i^{\frac{1}{2}}}{p_i^2-m_i^2+i\epsilon}\,. \quad (21)$$

Thus we obtain from Eqs.(16,17,21) [8]

$$<j_s\cdots|S|\cdots i_s>=\,\bar{u}_j^s\,\bar{Z}_j^{\frac{1}{2}}\mathcal{M}^{amp}(i_s\cdots\to\cdots j_s)Z_i^{\frac{1}{2}}u_i^s\,. \quad (22)$$

This is just the usual form of the S-matrix elements containing two external-line fermions [9].

From Eq.(20) the fermion WRC can be obtain by expanding the fermion propagation amplitude at $p^2\to m_i^2$. Like the boson's case, the fermion WRC becomes [8]

$$\begin{aligned}\bar{Z}_i^{L\frac{1}{2}}Z_i^{L\frac{1}{2}} &= (1+\Sigma_{ii}^R(m_i^2))A\,,\\ \bar{Z}_i^{R\frac{1}{2}}Z_i^{R\frac{1}{2}} &= (1+\Sigma_{ii}^L(m_i^2))A\,,\\ \bar{Z}_i^{L\frac{1}{2}}Z_i^{R\frac{1}{2}} &= Z_i^{L\frac{1}{2}}\bar{Z}_i^{R\frac{1}{2}}\,,\end{aligned} \quad (23)$$

where the fermion self energy is written as [8]

$$\Sigma_{ii}(\not{p}) = \not{p}\gamma_L\Sigma_{ii}^L(p^2)+\not{p}\gamma_R\Sigma_{ii}^R(p^2)+m_i\Sigma_{ii}^S(p^2)\,, \quad (24)$$

and

$$A = \Big(1+\big[\Sigma_{ii}^L+\Sigma_{ii}^R+\Sigma_{ii}^L\Sigma_{ii}^R+m_i^2(\Sigma_{ii}^{L\prime}+\Sigma_{ii}^{R\prime}+\Sigma_{ii}^{L\prime}\Sigma_{ii}^R+\Sigma_{ii}^L\Sigma_{ii}^{R\prime})+2m_i(m_i+\delta m_i-m_i\Sigma_{ii}^S)\Sigma_{ii}^{S\prime}\big]_{p^2=m_i^2}\Big)^{-1}\,. \quad (25)$$

On the other hand, from the relationship between the two matrix elements: $<\lambda_{i_s}|\bar{\psi}_i(0)|\Omega>=<\Omega|\psi_i(0)|\lambda_{i_s}>^\dagger\,\gamma_0$ we also obtain a hermitian conjugation relationship between the incoming and outgoing fermion WRC (see Eqs.(17,18))

$$\bar{Z}_i^{L\frac{1}{2}} = Z_i^{L\frac{1}{2}*}\,,\qquad \bar{Z}_i^{R\frac{1}{2}} = Z_i^{R\frac{1}{2}*}\,. \quad (26)$$

This coincides with the constraint of Eq.(3) for fermion FRC. But such hermitian conjugation relationship has been broken by the imaginary parts of unstable fermion's propagation amplitudes [1, 2, 5]. Besides, the result of Eqs.(23) also shows that this hermitian conjugation relationship isn't satisfied by the fermion WRC (see appendix A).

III. DETERMINATION OF THE WAVE-FUNCTION RENORMALIZATION CONSTANTS

In fact there are also two set WRC for antiparticles. For anti-boson WRC we can introduce them as follows:

$$\int d^4x\, e^{i\,p_i\cdot x}\cdots\int d^4y\, e^{-i\,p_j\cdot y} <\Omega|T\{\phi_j(y)\cdots\phi_i^\dagger(x)\}|\Omega>\sim \frac{i<\Omega|\phi_i^\dagger(0)|\lambda_{\bar{i}}>}{p_i^2-m_i^2+i\epsilon}\cdots\frac{i<\lambda_{\bar{j}}|\phi_j(0)|\Omega>}{p_j^2-m_j^2+i\epsilon}<\bar{i}\cdots|S|\cdots\bar{j}>\,, \quad (27)$$

where $\lambda_{\bar{i}}$ and $\lambda_{\bar{j}}$ are the outgoing and incoming anti-boson's states of S-matrix elements, and

$$Z_{\bar{i}}^{\frac{1}{2}}\ =<\Omega|\phi_i^\dagger(0)|\lambda_{\bar{i}}>\,,\qquad \bar{Z}_{\bar{i}}^{\frac{1}{2}}\ =<\lambda_{\bar{i}}|\phi_i(0)|\Omega>\ . \quad (28)$$

The anti-boson's propagation amplitude in interaction vacuum thus is

$$\int d^4x\, e^{i\,p\cdot x}<\Omega|T\{\phi_i(0)\phi_i^\dagger(x)\}|\Omega>\sim\ \frac{i<\Omega|\phi_i^\dagger(0)|\lambda_{\bar{i}}><\lambda_{\bar{i}}|\phi_i(0)|\Omega>}{p^2-m_i^2+i\epsilon}\ =\ \frac{i\,Z_{\bar{i}}^{\frac{1}{2}}\bar{Z}_{\bar{i}}^{\frac{1}{2}}}{p^2-m_i^2+i\epsilon}\,. \quad (29)$$

If the charge conjugation is conserved, we will have from Eqs.(9,28)

$$Z_i^{\frac{1}{2}} = Z_{\bar{i}}^{\frac{1}{2}}\,,\qquad \bar{Z}_i^{\frac{1}{2}} = \bar{Z}_{\bar{i}}^{\frac{1}{2}}\,. \quad (30)$$

On the other hand, if the charge conjugation isn't conserved, the only variation in standard model is the coupling constants change from real numbers into complex numbers. But in the present problem such variation also disappears, because the Feynman diagrams which generate the matrix elements of Eqs.(9,28) are symmetric in incoming and outgoing states, thus the products of all the coupling constants only include the module squares of the complex coupling constants like the case of the charge conjugation conservation. So Eqs.(30) also holds true in standard model.

Besides, the boson WRC and the anti-boson WRC can also be related by the CPT conservation law. From Eq.(29), Eq.(27) can also be written as

$$\int d^4x\, e^{i\,p_i\cdot x}\cdots\int d^4y\, e^{-i\,p_j\cdot y} <\Omega|T\{\phi_j(y)\cdots\phi_i^\dagger(x)\}|\Omega>\sim \frac{i\,Z_{\bar{i}}^{\frac{1}{2}}\bar{Z}_{\bar{i}}^{\frac{1}{2}}}{p_i^2-m_i^2+i\epsilon}\cdots\frac{i\,Z_{\bar{j}}^{\frac{1}{2}}\bar{Z}_{\bar{j}}^{\frac{1}{2}}}{p_j^2-m_j^2+i\epsilon}\mathcal{M}^{amp}(\bar{j}\cdots\to\cdots\bar{i})\,. \quad (31)$$

From Eqs.(27,28,31) we obtain

$$<\bar{i}\cdots|S|\cdots\bar{j}>=\,Z_{\bar{j}}^{\frac{1}{2}}\mathcal{M}^{amp}(\bar{j}\cdots\to\cdots\bar{i})\bar{Z}_{\bar{i}}^{\frac{1}{2}}\,. \quad (32)$$

According to the CPT conservation law for bosons

$$<j\cdots|S|\cdots i>=<\bar{i}\cdots|S|\cdots\bar{j}>\,, \quad (33)$$

and the fact (i and j are bosons)

$$\mathcal{M}^{amp}(i\cdots\to\cdots j)\;=\;\mathcal{M}^{amp}(\bar{j}\cdots\to\cdots\bar{i})\,, \quad (34)$$

we obtain (see Eqs.(12,32))

$$Z_{\bar{j}}^{\frac{1}{2}}\;=\;\bar{Z}_j^{\frac{1}{2}}\,,\qquad \bar{Z}_{\bar{i}}^{\frac{1}{2}}\;=\;Z_i^{\frac{1}{2}}\,. \quad (35)$$

Since i and j are arbitrary bosons, we finally obtain (see Eqs.(14,30,35))

$$\bar{Z}_i\;=\;Z_i\;=\;\bar{Z}_{\bar{i}}\;=\;Z_{\bar{i}}\;=\;(1+\Sigma'_{ii}(m_i^2))^{-1}\,. \quad (36)$$

Similarly we can get the formula for fermion WRC. For the Green function with two anti-fermion external lines we have

$$\int d^4x\, e^{i\,p_i\cdot x}\cdots\int d^4y\, e^{-i\,p_j\cdot y} <\Omega|T\{\psi_j(y)\cdots\bar{\psi}_i(x)\}|\Omega>\sim -\frac{i<\lambda_{\bar{j}_s}|\psi_j(0)|\Omega>}{p_j^2-m_j^2+i\epsilon}<\bar{i}_s\cdots|S|\cdots\bar{j}_s>\frac{i<\Omega|\bar{\psi}_i(0)|\lambda_{\bar{i}_s}>}{p_i^2-m_i^2+i\epsilon}\,, \quad (37)$$

where $x^0>y^0$, so the time-ordering operator T exchanges the positions of the fermion fields $\psi_j(y)$ and $\bar{\psi}_i(x)$, thus a minus sign appears. Note that the order of the spins and the gamma matrices keeps unchanged. The anti-fermion WRC can be introduced as follows:

$$<\Omega|\bar{\psi}_i(0)|\lambda_{\bar{i}_s}>=\,\bar{\nu}_i^s\,Z_{\bar{i}}^{\frac{1}{2}}\,,\qquad <\lambda_{\bar{i}_s}|\psi_i(0)|\Omega>=\,\bar{Z}_{\bar{i}}^{\frac{1}{2}}\,\nu_i^s\,, \quad (38)$$

with

$$Z_{\bar{i}}^{\frac{1}{2}}\;=\;Z_{\bar{i}}^{L\frac{1}{2}}\gamma_R+Z_{\bar{i}}^{R\frac{1}{2}}\gamma_L\,,\qquad \bar{Z}_{\bar{i}}^{\frac{1}{2}}\;=\;\bar{Z}_{\bar{i}}^{L\frac{1}{2}}\gamma_L+\bar{Z}_{\bar{i}}^{R\frac{1}{2}}\gamma_R\,. \quad (39)$$

The anti-fermion propagation amplitude thus is

$$\int d^4x\, e^{i\,p\cdot x} <\Omega|T\{\psi_i(0)\bar{\psi}_i(x)\}|\Omega>\sim -\frac{\sum_s i<\lambda_{\bar{i}_s}|\psi_i(0)|\Omega><\Omega|\bar{\psi}_i(0)|\lambda_{\bar{i}_s}>}{p^2-m_i^2+i\epsilon}\;=\;-\frac{i\,\bar{Z}_{\bar{i}}^{\frac{1}{2}}\sum_s\nu_i^s\,\bar{\nu}_i^s\,Z_{\bar{i}}^{\frac{1}{2}}}{p^2-m_i^2+i\epsilon}\,. \quad (40)$$

If the charge conjugation is conserved, according to the formula $u_i^s(p)=-i\gamma_2(\nu_i^s(p))^*$ we have from Eqs.(17,18)

$$\begin{aligned}
<\Omega|\bar{\psi}_i(0)|\lambda_{\bar{i}_s}> \;&=\; -i<\Omega|\psi_i(0)\gamma_2|\lambda_{i_s}>^T\gamma_0\;=\;-i\left(Z_i^{\frac{1}{2}}\gamma_2\,u_i^s\right)^T\gamma_0\;=\;\bar{\nu}_i^s\left(Z_i^{L\frac{1}{2}}\gamma_R+Z_i^{R\frac{1}{2}}\gamma_L\right)\,,\\
<\lambda_{\bar{i}_s}|\psi_i(0)|\Omega> \;&=\; i\,\gamma_0<\lambda_{i_s}|\gamma_2\,\bar{\psi}_i(0)|\Omega>^T=\,i\gamma_0\left(\bar{u}_i^s\gamma_2\,\bar{Z}_i^{\frac{1}{2}}\right)^T\;=\;\left(\bar{Z}_i^{L\frac{1}{2}}\gamma_L+\bar{Z}_i^{R\frac{1}{2}}\gamma_R\right)\nu_i^s\,,
\end{aligned} \quad (41)$$

where the transposition operator T only transposes the spins and the gamma matrices. From Eqs.(38,39) it is obvious that

$$\begin{aligned} Z_{\bar{i}}^{L\frac{1}{2}} &= Z_i^{L\frac{1}{2}}\,, \qquad & Z_{\bar{i}}^{R\frac{1}{2}} &= Z_i^{R\frac{1}{2}}\,, \\ \bar{Z}_{\bar{i}}^{L\frac{1}{2}} &= \bar{Z}_i^{L\frac{1}{2}}\,, \qquad & \bar{Z}_{\bar{i}}^{R\frac{1}{2}} &= \bar{Z}_i^{R\frac{1}{2}}\,. \end{aligned} \tag{42}$$

Although in standard model the charge conjugation isn't conserved, Eqs.(41) also keeps unchanged because of the same reason as boson's: the Feynman diagrams which generate the matrix elements of Eqs.(17,38) are symmetric in incoming and outgoing states thus the products of all the coupling constants only include the module squares of the complex coupling constants like the case of the charge conjugation conservation. So Eqs.(42) also hold true in standard model.

Besides, we can also relate the anti-fermion WRC to fermion WRC by the CPT conservation law. From Eq.(40), Eq.(37) can also be written as

$$\int d^4x\, e^{i\,p_i\cdot x}\cdots\int d^4y\, e^{-i\,p_j\cdot y} <\!\Omega|T\{\psi_j(y)\cdots\bar{\psi}_i(x)\}|\Omega\!> \sim -\frac{i\,\bar{Z}_{\bar{j}}^{\frac{1}{2}}\,\nu_j^s\,\bar{\nu}_j^s\,Z_{\bar{j}}^{\frac{1}{2}}}{p_j^2-m_j^2+i\epsilon}\mathcal{M}^{amp}(\bar{j}_s\cdots\to\cdots\bar{i}_s)\frac{i\,\bar{Z}_{\bar{i}}^{\frac{1}{2}}\,\nu_i^s\,\bar{\nu}_i^s\,Z_{\bar{i}}^{\frac{1}{2}}}{p_i^2-m_i^2+i\epsilon}\,. \tag{43}$$

From Eqs.(37,38,43) we obtain

$$<\!\bar{i}_s\cdots|S|\cdots\bar{j}_s\!> = \bar{\nu}_j^s\,Z_{\bar{j}}^{\frac{1}{2}}\mathcal{M}^{amp}(\bar{j}_s\cdots\to\cdots\bar{i}_s)\bar{Z}_{\bar{i}}^{\frac{1}{2}}\nu_i^s\,. \tag{44}$$

From the CPT conservation law for fermions

$$<\!j_s\cdots|S|\cdots i_s\!> = -<\!\bar{i}_s\cdots|S|\cdots\bar{j}_s\!>\,, \tag{45}$$

and Eqs.(22,44) we have

$$\bar{u}_j^s\,\bar{Z}_j^{\frac{1}{2}}\mathcal{M}^{amp}(i_s\cdots\to\cdots j_s)Z_i^{\frac{1}{2}}u_i^s = -\bar{\nu}_j^s\,Z_{\bar{j}}^{\frac{1}{2}}\mathcal{M}^{amp}(\bar{j}_s\cdots\to\cdots\bar{i}_s)\bar{Z}_{\bar{i}}^{\frac{1}{2}}\nu_i^s\,. \tag{46}$$

We can decompose $\mathcal{M}^{amp}$ into its most general Dirac structure

$$\begin{aligned} \mathcal{M}^{amp}(i_s(p_i)\cdots\to\cdots j_s(p_j)) &= a(q^2)\,\not{q}\,\gamma_L+b(q^2)\,\not{q}\,\gamma_R+c(q^2)\,\not{p}\,\gamma_L+d(q^2)\,\not{p}\,\gamma_R+e(q^2)\gamma_L+f(q^2)\gamma_R\,, \\ \mathcal{M}^{amp}(\bar{j}_s(p_j)\cdots\to\cdots\bar{i}_s(p_i)) &= -a(q^2)\,\not{q}\,\gamma_L-b(q^2)\,\not{q}\,\gamma_R-c(q^2)\,\not{p}\,\gamma_L-d(q^2)\,\not{p}\,\gamma_R+e(q^2)\gamma_L+f(q^2)\gamma_R\,, \end{aligned} \tag{47}$$

with

$$p = p_i+p_j\,, \qquad q = p_i-p_j\,. \tag{48}$$

Putting Eqs.(47) into Eq.(46) we obtain

$$\begin{aligned} &-a(q^2)(m_jA_L-m_iA_R)Z_i^{L\frac{1}{2}}\bar{Z}_j^{L\frac{1}{2}}+b(q^2)(m_iA_L-m_jA_R)Z_i^{R\frac{1}{2}}\bar{Z}_j^{R\frac{1}{2}}+c(q^2)(m_jA_L+m_iA_R)Z_i^{L\frac{1}{2}}\bar{Z}_j^{L\frac{1}{2}} \\ &+d(q^2)(m_iA_L+m_jA_R)Z_i^{R\frac{1}{2}}\bar{Z}_j^{R\frac{1}{2}}+e(q^2)A_LZ_i^{L\frac{1}{2}}\bar{Z}_j^{R\frac{1}{2}}+f(q^2)A_RZ_i^{R\frac{1}{2}}\bar{Z}_j^{L\frac{1}{2}} \\ =\; & a(q^2)(m_jB_L-m_iB_R)\bar{Z}_{\bar{i}}^{L\frac{1}{2}}Z_{\bar{j}}^{L\frac{1}{2}}-b(q^2)(m_iB_L-m_jB_R)\bar{Z}_{\bar{i}}^{R\frac{1}{2}}Z_{\bar{j}}^{R\frac{1}{2}}-c(q^2)(m_jB_L+m_iB_R)\bar{Z}_{\bar{i}}^{L\frac{1}{2}}Z_{\bar{j}}^{L\frac{1}{2}} \\ &-d(q^2)(m_iB_L+m_jB_R)\bar{Z}_{\bar{i}}^{R\frac{1}{2}}Z_{\bar{j}}^{R\frac{1}{2}}-e(q^2)B_L\bar{Z}_{\bar{i}}^{L\frac{1}{2}}Z_{\bar{j}}^{R\frac{1}{2}}-f(q^2)B_R\bar{Z}_{\bar{i}}^{R\frac{1}{2}}Z_{\bar{j}}^{L\frac{1}{2}}\,, \end{aligned} \tag{49}$$

with

$$\begin{aligned} A_L &= \bar{u}_j^s(p_j)\gamma_L\,u_i^s(p_i)\,, \qquad & A_R &= \bar{u}_j^s(p_j)\gamma_R\,u_i^s(p_i)\,, \\ B_L &= \bar{\nu}_j^s(p_j)\gamma_L\,\nu_i^s(p_i)\,, \qquad & B_R &= \bar{\nu}_j^s(p_j)\gamma_R\,\nu_i^s(p_i)\,. \end{aligned} \tag{50}$$

Using the fact that

$$A_L = -B_L\,, \qquad A_R = -B_R\,, \tag{51}$$

we easily obtain from Eq.(49)

$$\begin{aligned} Z_i^{L\frac{1}{2}} &= \bar{Z}_{\bar{i}}^{L\frac{1}{2}}\,, \qquad & Z_i^{R\frac{1}{2}} &= \bar{Z}_{\bar{i}}^{R\frac{1}{2}}\,, \\ \bar{Z}_j^{L\frac{1}{2}} &= Z_{\bar{j}}^{L\frac{1}{2}}\,, \qquad & \bar{Z}_j^{R\frac{1}{2}} &= Z_{\bar{j}}^{R\frac{1}{2}}\,. \end{aligned} \tag{52}$$

Combining Eqs.(23,42,52) we finally obtain

$$\bar{Z}_i^L = Z_i^L = \bar{Z}_{\bar{i}}^L = Z_{\bar{i}}^L = (1+\Sigma_{ii}^R(m_i^2))A\,,$$
$$\bar{Z}_i^R = Z_i^R = \bar{Z}_{\bar{i}}^R = Z_{\bar{i}}^R = (1+\Sigma_{ii}^L(m_i^2))A\,. \tag{53}$$

We note that a similar result is firstly proposed in Ref.[8] as an assumption. Here we derive it under a rational foundation.

Now we have totally determined the diagonal boson and fermion WRC. All of the one-loop results of the WRC are listed in the appendix B. There are also off-diagonal WRC, but they are different from the diagonal WRC under the LSZ reduction formula. We note that the off-diagonal WRC should be determined by the prescriptions in Ref.[2, 8].

IV. GAUGE DEPENDENCE OF PHYSICAL AMPLITUDES UNDER THE CONVENTIONAL AND THE PRESENT WAVE-FUNCTION RENORMALIZATION PRESCRIPTION

In order to investigate whether the present wave-function renormalization prescription is rational, we calculate two physical processes to see if the physical amplitudes keep gauge invariant under the present wave-function renormalization prescription.

Firstly we discuss the physical process $W^+ \to u_i\bar{d}_j$, i.e. the gauge boson W decaying into up-type i and down-type j quarks. At one-loop level Eqs.(53) are equivalent to Eqs.(4.10) of Ref.[2] if set $\alpha_i = 0$. Using the Nielsen identities [10] Espriu et al. have proved that the physical amplitude $\mathcal{M}(W^+ \to u_i\bar{d}_j)$ is gauge independent under the present wave-function renormalization prescription [2].

Secondly we discuss the physical process $Z \to d_i\bar{d}_i$, i.e. the gauge boson Z decaying into a pair of down-type i quarks. Our numerical calculation has demonstrated the real part of the physical amplitude is gauge-parameter independent, so we only need to discuss the gauge dependence of the imaginary part of the physical amplitude. At one-loop level we have

$$\mathcal{M}(Z\to d_i\bar{d}_i) = \frac{e(2c_W^2+1)}{12c_W s_W}\left(\delta Z_Z+\delta\bar{Z}_{d_i}^L+\delta\bar{Z}_{\bar{d}_i}^L\right)A_L - \frac{e\,s_W}{6c_W}\left(\delta Z_Z+\delta\bar{Z}_{d_i}^R+\delta\bar{Z}_{\bar{d}_i}^R\right)A_R + \mathcal{M}^{amp}(Z\to d_i\bar{d}_i)\,, \tag{54}$$

where e is electron charge, s_W and c_W are the sine and cosine of the weak mixing angle, and

$$A_L = \bar{u}(p_{d_i})\not{\epsilon}\gamma_L\nu(p_{\bar{d}_i})\,, \qquad A_R = \bar{u}(p_{d_i})\not{\epsilon}\gamma_R\nu(p_{\bar{d}_i})\,, \tag{55}$$

and $\mathcal{M}^{amp}(Z\to d_i\bar{d}_i)$ is the amplitude of the amputated diagrams shown in Fig.1. Using the *cutting rules* [11] we obtain

$$\begin{aligned}
Im\mathcal{M}^{amp}(Z\to d_i\bar{d}_i)|_\xi \;=\;& A_L(2c_W^2+1)\Big[\frac{e^3}{1152\pi\, c_W^3 s_W^3}(1-4c_W^2\xi_W)^{3/2}\theta[M_Z-2\sqrt{\xi_W}M_W] \\
&- \frac{e^3}{192\pi\, c_W s_W^3\, x_{d,i}}\sum_j |V_{ji}|^2(x_{d,i}-x_{u,j}-\xi_W)\,B\,\theta[m_{d,i}-m_{u,j}-\sqrt{\xi_W}M_W] \\
&- \frac{e^3}{576\pi\, c_W^3 s_W}\left((\xi_W-1)^2c_W^4-2(\xi_W-5)c_W^2+1\right)C\,\theta[M_Z-M_W-\sqrt{\xi_W}M_W]\Big] \\
&+ A_R\Big[-\frac{e^3}{576\pi\, c_W^3 s_W}(1-4c_W^2\xi_W)^{3/2}\theta[M_Z-2\sqrt{\xi_W}M_W] \\
&+ \frac{s_W\, e^3}{288\pi\, c_W^3}\left((\xi_W-1)^2c_W^4-2(\xi_W-5)c_W^2+1\right)C\,\theta[M_Z-M_W-\sqrt{\xi_W}M_W]\Big]\,,
\end{aligned} \tag{56}$$

where the subscript ξ takes the gauge-parameter-dependent part, V_{ji} is the CKM matrix element [12], M_W and ξ_W are the mass and gauge parameter of gauge boson W, M_Z is the mass of gauge boson Z, $m_{d,i}$ and $m_{u,j}$ are the masses of d_i quark and up-type j quark, and $x_{d,i} = m_{d,i}^2/M_W^2$, $x_{u,j} = m_{u,j}^2/M_W^2$, and

$$B = \sqrt{\xi_W^2-2(x_{d,i}+x_{u,j})\xi_W+(x_{d,i}-x_{u,j})^2}\,, \qquad C = \sqrt{(\xi_W-1)^2c_W^4-2(\xi_W+1)c_W^2+1}\,. \tag{57}$$

We note that the result of Eq.(56) coincides with the results of the conventional loop momentum integral algorithm [11] and the causal perturbative theory [13].

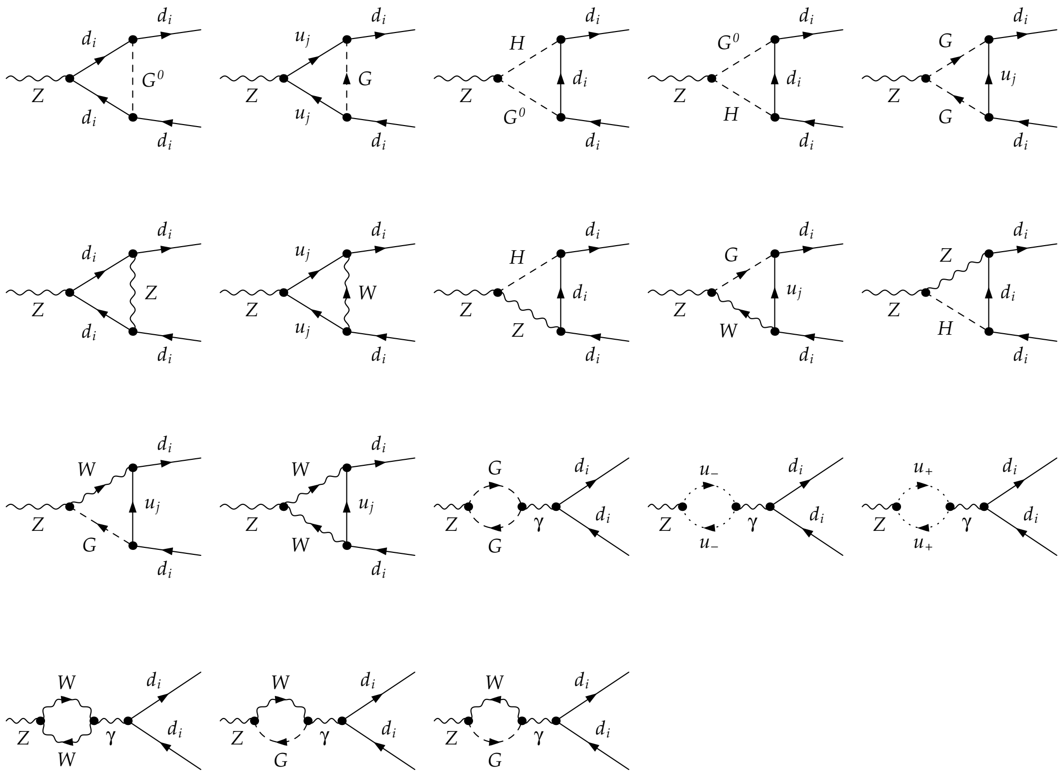

FIG. 1: One-loop gauge-parameter-dependent diagrams of $Z \to d_i \bar{d}_i$ which contain imaginary-part contribution.

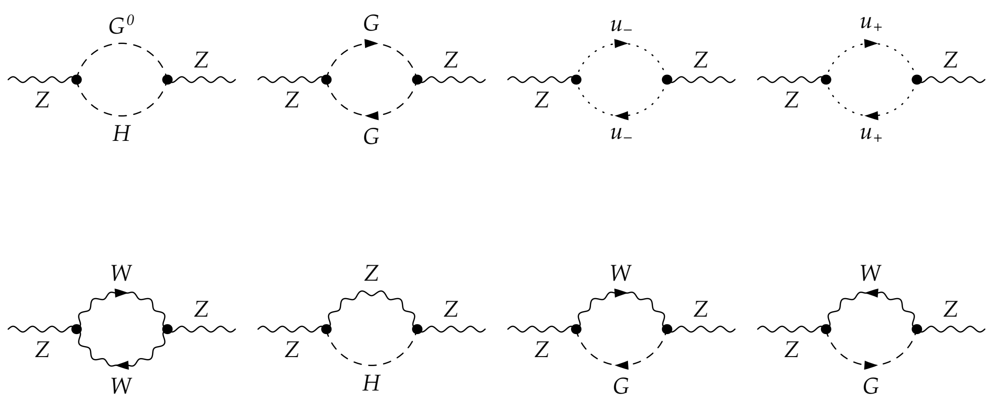

FIG. 2: One-loop gauge-parameter-dependent Z self-energy diagrams which contain imaginary-part contribution.

Then we calculate the WRC of gauge boson Z and d_i quark. In Fig.2 we show the one-loop Z self-energy diagrams which are used to calculate the gauge-parameter-dependent imaginary part of δZ_Z. Using Eqs.(36) and the *cutting rules* we obtain

$$\begin{aligned} Im\,\delta Z_Z|_\xi &= -\frac{e^2}{96\pi\, c_W^2 s_W^2}(1-4c_W^2\xi_W)^{3/2}\theta[M_Z - 2\sqrt{\xi_W}M_W] \\ &+ \frac{e^2}{48\pi\, c_W^2}\left((\xi_W-1)^2 c_W^4 - 2(\xi_W-5)c_W^2 + 1\right)\, C\,\theta[M_Z - M_W - \sqrt{\xi_W}M_W]\,. \end{aligned} \tag{58}$$

The d_i self-energy diagrams used to calculate the gauge-parameter-dependent imaginary part of d_i WRC have been shown in Fig.3. Using Eqs.(53) and the *cutting rules* we obtain

$$Im\,\delta\bar{Z}^R_{d_i}|_\xi \;=\; Im\,\delta\bar{Z}^R_{\bar{d}_i}|_\xi \;=\; 0\,,$$

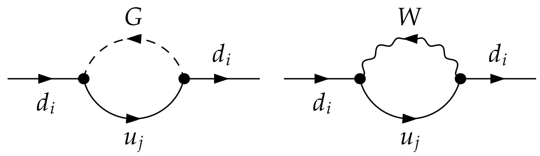

FIG. 3: One-loop d_i self-energy diagrams which contain imaginary-part contribution.

$$Im\,\delta\bar{Z}^L_{d_i}|_\xi \;=\; Im\,\delta\bar{Z}^L_{\bar{d}_i}|_\xi \;=\; \frac{e^2}{32\pi\, s_W^2\, x_{d,i}}\sum_j |V_{ji}|^2(x_{d,i}-x_{u,j}-\xi_W)\,B\,\theta[m_{d,i}-m_{u,j}-\sqrt{\xi_W}M_W]\,. \qquad (59)$$

Put Eqs.(56,58,59) into Eq.(54) we finally obtain

$$Im\mathcal{M}(Z\to d_i\bar{d}_i)|_\xi \;=\; 0\,. \qquad (60)$$

This means the present wave-function renormalization prescription keeps the physical amplitude of $Z\to d_i\bar{d}_i$ gauge-parameter independent.

For comparison we evaluate the gauge dependence of physical amplitde $Z\to d_i\bar{d}_i$ under the conventional wave-function renormalization prescription which discards the imaginary part of unstable particle's WRC [4]. Under the conventional wave-function renormalization prescription only the last term of the r.h.s. of Eq.(54) contributes to the imaginary part of $\mathcal{M}(Z\to d_i\bar{d}_i)$. According to Eqs.(54,56) one readily has

$$\begin{aligned}
Im\mathcal{M}(Z\to d_i\bar{d}_i)|_\xi \;=\;& A_L(2c_W^2+1)\left[\frac{e^3}{1152\pi\, c_W^3 s_W^3}(1-4c_W^2\xi_W)^{3/2}\theta[M_Z-2\sqrt{\xi_W}M_W]\right.\\
&-\;\frac{e^3}{192\pi\, c_W s_W^3\, x_{d,i}}\sum_j |V_{ji}|^2(x_{d,i}-x_{u,j}-\xi_W)\,B\,\theta[m_{d,i}-m_{u,j}-\sqrt{\xi_W}M_W]\\
&\left.-\;\frac{e^3}{576\pi\, c_W^3 s_W}\left((\xi_W-1)^2c_W^4-2(\xi_W-5)c_W^2+1\right)\,C\,\theta[M_Z-M_W-\sqrt{\xi_W}M_W]\right]\\
&+\;A_R\left[-\frac{e^3}{576\pi\, c_W^3 s_W}(1-4c_W^2\xi_W)^{3/2}\theta[M_Z-2\sqrt{\xi_W}M_W]\right.\\
&\left.+\;\frac{s_W\, e^3}{288\pi\, c_W^3}\left((\xi_W-1)^2c_W^4-2(\xi_W-5)c_W^2+1\right)\,C\,\theta[M_Z-M_W-\sqrt{\xi_W}M_W]\right]\,. \qquad (61)
\end{aligned}$$

This clearly proves that the conventional wave-function renormalization prescription which discards the imaginary part of unstable particle's WRC makes physical amplitude gauge dependent.

Through the two examples we can see the present wave-function renormalization prescription keeps physical amplitude gauge invariant, while other possible wave-function renormalization prescriptions, e.g. the prescriptions in Ref.[4], destroys the gauge invariance of physical amplitude. We note that the breaking of the gauge invariance of physical amplitude will break the gauge invariance of physical results.

V. CONCLUSION

We have discussed how to define and totally determine unstable particle's WRC under the postulation of the generalization of the LSZ reduction formula to unstable particles. We introduce two set particle's WRC, and find there are hermitian conjugation relationships between them. But such hermitian conjugation relationships have been broken by the imaginary parts of unstable particle's propagation amplitudes. By introducing two set antiparticle's WRC and the CPT conservation law we find a new wave-function renormalization condition which has been used to totally determine unstable particle's WRC. We have calculated two physical processes to demonstrate the consistence of the present wave-function renormalization prescription with the gauge theory in standard model. We also prove that the conventional wave-function renormalization prescription which discards the imaginary part of unstable particle's WRC makes physical amplitude gauge dependent.

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Appendix A

In this appendix we list the evidence that the present wave-function renormalization prescription of Eqs.(23) breaks the hermitian conjugation relationship of Eqs.(26). We calculate the imaginary part of down-type i quark's self energy (the corresponding diagrams have been shown in Fig.3) to demonstrate this problem. At one-loop level Eqs.(23) requires for d_i quark

$$Im\bar{Z}_{d,i}^{L\frac{1}{2}} Z_{d,i}^{L\frac{1}{2}} = \frac{e^2}{32\pi s_W^2 x_{d,i}} \sum_j |V_{ji}|^2 (x_{d,i} - x_{u,j} - \xi_W)\, B\, \theta[m_{d,i} - m_{u,j} - \sqrt{\xi_W} M_W] \,, \tag{62}$$

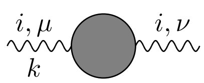
with B shown in Eqs.(57). Obviously this result contradicts Eqs.(26).

Appendix B

In this appendix we list all of the one-loop results of boson and fermion WRC. For boson we have from Eqs.(36)

$$\bar{Z}_i = Z_i = \bar{Z}_{\bar{i}} = Z_{\bar{i}} = 1 - \frac{\partial}{\partial p^2}\Sigma_{ii}(m_i^2) \,. \tag{63}$$

For vector boson Σ_{ii} is the transverse part of its self energy, i.e. Σ_{ii}^T in the following equation:



$$= -ig^{\mu\nu}(k^2 - m_i^2) - i(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2})\Sigma_{ii}^T(k^2) - i\frac{k^\mu k^\nu}{k^2}\Sigma_{ii}^L(k^2) \,. \tag{64}$$

For fermion we have from Eqs.(53)

$$\begin{aligned} \bar{Z}_i^L &= Z_i^L = \bar{Z}_{\bar{i}}^L = Z_{\bar{i}}^L = 1 - \Sigma_{ii}^L(m_i^2) - m_i^2 \frac{\partial}{\partial p^2}\left(\Sigma_{ii}^L(p^2) + \Sigma_{ii}^R(p^2) + 2\Sigma_{ii}^S(p^2)\right)_{p^2=m_i^2} \,, \\ \bar{Z}_i^R &= Z_i^R = \bar{Z}_{\bar{i}}^R = Z_{\bar{i}}^R = 1 - \Sigma_{ii}^R(m_i^2) - m_i^2 \frac{\partial}{\partial p^2}\left(\Sigma_{ii}^L(p^2) + \Sigma_{ii}^R(p^2) + 2\Sigma_{ii}^S(p^2)\right)_{p^2=m_i^2} \,. \end{aligned} \tag{65}$$

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